Synthesis of model predictive controller for an identified model of MIMO process

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Article Info	**ABSTRACT**

Model Predictive Controller (MPC) technology has been researched and developed to meet varied demands of need to control industrial power plants and petroleum refineries. This development has paved the way for the MPC technology too many other fields like automotive, aerospace, food processing industries in this paper, primary importance has been paid to the development of a MPC for an identified model of Multiple Input and Multiple Output process. In this paper, a Four Tank System has been considered for generation of input-output data. This data i.e. generated input output data is used for the estimation of two polynomial model, name ARX model (Autoregressive exogenous) model and OE (Output Error) model. With each of model output generated, the Fit-Rates of models are compared to find out most efficient model. The model equations are now considered as plant for developing a Model Predictive Controller (MPC). Two sets of results are obtained after the development of MPC and tested. One is without noise and one is with noise. Both sets of results were a success as the output signals traces step input signals after some steady oscillations in real time with in a very short period of time which indicated a good response time. The MPC developed can be applied to any polynomial model with a good Fit-Rate, it predicts and control the process variables automatically.

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1. INTRODUCTION

Model identification of a single input and single output system is obtained by minimizing the output error of a closed loop model using constraint optimization. The optimization program is implemented by using FOMCON tool in Matlab [1]. A systematic procedure was derived for handling stationary random problems with multiple inputs and multiple outputs where only auto-spectra are known. This method can also be applied to nonstationary random inputs and deterministic sine and transient problems [2]. Accurate models were derived from the both physical and experimental data and decentralized control was demonstrated on the process. The main purpose was to provide connection between the abstract control theory and the real world. The quadruple-tank process well suited to illustrate performance limitations in multivariable control design caused by right half plane zeros [3]. A mathematically clean formulation of MPC emerged which allowed many researchers to address problems like feasibility stability, performance in a rigorous manner. In nonlinear area variety of issues remained which are technically complex but have potentially significant practical implications for stability and performance and the computational complexity necessary to achieve them. Simulation results of continuous stirred tank heater (CSTR) open loop and closed loop are generated by using Simulink implementations Matlab [4]. MPC has the ability to anticipate future events and can take

control accordingly. A process model is used to predict the current values of the output variables. The residuals the differences between the actual and the predicted outputs, serve as the feedback signal to a prediction block The predictions are used in two types of MPC calculations that are performed at each sampling instant: set point calculations and the control calculations. The coordination of the control and the set-point calculations is a unique feature of MPC [5] Optimization plays a significant role in designing process of MPC. FARX model of thermal dynamics of buildings is identified by using least square parameter estimation method and Compared the FARX model with the integer order ARX model [6-8]. Identified FARX model is validated by using IESVE software [6]. MPC is proposed to design the energy management system for providing reliable and optimized production of different sources of microgrid [9]. The offline MPC controller based on MPP method was offered for secondary loop to allow motor current to imitate the reference signal from the primary controller. Optimization problem in the MPC controller was altered by incorporating a Laguerre Model approach to decrease the number of decision variables [10].

The detailed above discussion is a stimulus to our present work. Modelling and controlling of multivariable systems is very tedious because they are highly interactive and complex [11]. Often complex models are beneficial for the analysis of system behavior whereas simple models are generally used for design of feedback controllers [12]. Design of controller system for any process to enhance the cost effective modelling and controlling is a difficult task [13]. The entire paper has been divided into three main parts namely, Data Generation [14], System Identification and Model Predictive Control. Basically, data generation refers to process of generating test data [15] for purpose of estimation and testing activities. The data needs to be generated in adherence to the set up test environment and should be in synchronization with developed test cases to fulfil requirement of testing model. Finally data is generated manually by considering the plant equations of Quadruple Tank System. System identification uses the input and output signals that are measured from a system to estimate the values of adjustable parameters in a given model structure. Model Predictive Control, advanced method of process control that has been in use in the process industries in chemical plants and oil refineries. They rely on dynamic model of the process, most often many linear empirical models obtained by system identification.

2.   RESEARCH METHOD

In this section, data generation from quadruple tank system is generated using System Identification Tool Box and some brief concepts of Model Predictive Controller(MPC) are discussed.

2.1. Physical Model

Quadruple tank consists of two inputs namely pump voltage v1, pump voltage v2 and two outputs namely tank1 height, tank 2 height. Schematic diagram of quadruple tank process as shown in Figure 1.
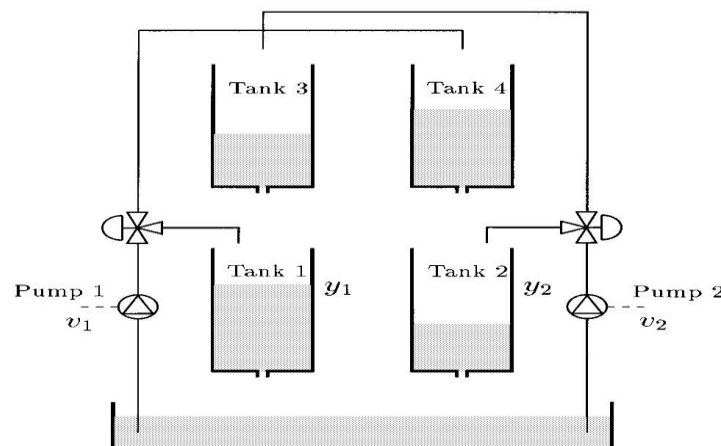

Figure 1. Schematic diagram of quadruple tank process [3]

The quadruple tank process is represented by following equations [3]:

$$\frac{dh1}{dt} = -\frac{a1}{A1}\sqrt{2gh1} + \frac{a3}{A1}\sqrt{2gh3} + \frac{\gamma 1k1}{A1}\upsilon_1 \qquad (1)$$

$$\frac{dh2}{dt} = -\frac{a2}{A2}\sqrt{2gh2} + \frac{a4}{A2}\sqrt{2gh4} + \frac{\gamma 2 k2}{A2}\upsilon_2 \tag{2}$$

$$\frac{dh3}{dt} = -\frac{a3}{A3}\sqrt{2gh3} + \frac{(1-\gamma 2)k2}{A3}\upsilon_2 \tag{3}$$

$$\frac{dh4}{dt} = -\frac{a4}{A4}\sqrt{2gh4} + \frac{(1-\gamma 1)k1}{A4}\upsilon_1 \tag{4}$$

Where h_i=level of tank i, A_i = cross section of tank i, a_i = cross section of outlet pipe.

The voltage applied to Pump is v_i and its flow is $k_i v_i$. The parameters γ_1, $\gamma_2 \in (0,1)$ are determined from how the valves are set prior to an experiment. The flow to Tank 1 is $\gamma 1 k 1 \upsilon 1$ and the flow to tank4 is $(1- \gamma 1) k1\upsilon1$ and similarly for Tanks 2 and 3. The acceleration of gravity is denoted by g and the measured level signals are $k_c h_1$ and $k_c h_2$. Out of all parameters, two inputs ($\upsilon 1$, $\upsilon 2$) and two outputs (h1, h2) are considered and plotted. During the data generation white noise unmeasured disturbance signals are added to the two outputs of the process. These unmeasured disturbance dynamics are assumed as the stochastic process of following governing models [16-17]

$$C(k) = \frac{0.04}{1-0.96z^{-1}} N_1(k) \tag{5}$$

$$C(k) = \frac{0.02}{1-0.97z^{-1}} N_2(k) \tag{6}$$

Training data is a dataset used to fit the parameter. Validation set can be regarded as a part of the training set because it is used to build model. It is also used for parameter selection and to avoid over-fitting. Out of 1000 samples 750 samples are taken for training data and 250 samples are taken for validation data.

Specifications:
System model parameters and initial conditions of quadruple tank assumed are as follows:
Model parameters of quadruple tank as shown in Table 1.

Table 1. Model Parameters of Quadruple Tank [3]

Parameters	Values
A1	28 cm^2
A2	32 cm^2
A3	28 cm^2
A4	32 cm^2
a1	0.07 cm^2
a2	0.057 cm^2
a3	0.07 cm^2
a4	0.057 cm^2
k_c	0.5V/cm
g	981 cm/sec^2
k1	3.33V/cm
k2	3.35V/cm
$\gamma 1$	0.7
$\gamma 2$	0.6

2.2. System Identification
2.2.1 Output Error Model (OE model): Structure of Output Error Model is Given by Following Equations [18-20]:

$$y(k)=B (q-1)/F (q-1) u(k) + e(k) \tag{7}$$

i.e., non-linear optimization problem for parameter estimation of output error defined.

$$\Psi (\hat{\upsilon}(1) \hat{\upsilon}(k)) = \sum_{i=1}^{N}[\hat{\upsilon}(k)]^2 \tag{8}$$

Where $\hat{\upsilon}(k) = y(k) - \hat{x}(k)$ $\tag{9}$

$$\hat{x}(k) = -a_1\hat{x}(k-1) - a_2\hat{x}(k-2) + b_1\hat{x}(k-2) + b_2\hat{x}(k-3) \tag{10}$$

2.2.2 Auto Regressive Exogenous Model (ARX model): The Equations for ARX [18, 21, 22] Model Given by Following:

$$y(k)=B (q-1)/A (q-1) u(k) + e(k) /A (q-1) \tag{11}$$

$$\hat{\theta} = \min_{a1, b1, a2, b2} \sum_{i=1}^{N}(\hat{e}(k))^{2} = \theta_{\min} (e^{T} *e) \tag{12}$$

$$\psi = e^{T}*e = [Y-\Omega\Theta]^{T} *[Y-\Omega\Theta] \tag{13}$$

2.2.3 Model Predictive Controller (MPC): The Conventional Kalman Filter [23] based on MPC Formulation is Developed Starting from Model:

$$x(k+1) =\Phi x(k)+\Gamma u(k)+w(k) \tag{14}$$

$$y(k) =C x(k)+ v(k) \tag{15}$$

$$\hat{x} (k+j+1|k) =\Phi\hat{x} (k+j|k) +\Gamma u(k+j|k) +L\varepsilon (k+j|k) \tag{16}$$

$$\varepsilon (k+j+1|k) = \varepsilon (k+j|k) \tag{17}$$

$$\hat{y}(k+j) =C\hat{x} (k+j|k) + \varepsilon (k+j|k) \tag{18}$$

For j=0,1,2…p-1. Here, ε belongs to R behaves as an integrated white noise signal that provides compensation for the unmeasured disturbance/model plant mismatch. The initial value at beginning of horizon, ε(k|k), is estimated by filtering the innovation sequence {e(k)} through a unity gain robustness filter [24] as follows:

$$e_{f}(k) = \phi_{e} e_{f}(k-1) +[1-\phi_{e}] e(k) \tag{19}$$

$$\varepsilon(k|k) =ef(k) \tag{20}$$

$$e(k) =y(k)-Cx(k|k-1) \tag{21}$$

$$\hat{x} (k|k-1) = \Phi\hat{x} (k-1|k-2) + \Gamma u(k-1) + Le(k-1) \tag{22}$$

To compare the performances of MPC controllers developed using identified models, the performance index (JMPC) defined as:

$$J_{MPC} = \sum_{k=1}^{Ns}[r(k) - y(k)]^{T} W_{E}[r(k)-y(k)] +\Delta u(k)^{T} W_{\Delta U} \Delta u(k) \text{ [7, 25]} \tag{23}$$

Is used which is based on objective function used in the proposed MPC formulation. In addition, the optimization technique used is integral square of errors (ISE) defined as:

$$ISE (yi) = \sum_{k=1}^{Ns}[yi(k) - ri(k)]^{2} \text{ [7]} \tag{24}$$

Where Ns is the length of simulation trail, r(k)-y(k) is an objective function, W_{E} is input weight, $W_{\Delta U}$ is the output weight.

3. RESULTS
3.1. Data Generation

In this section generation of stable response for quadruple tank system identification of efficient model among OE model & ARX model and generation of responses for MPC with and without noise are obtained. Input-output data is generated from MATLAB code of four tank system without noise which is represented in graphical forms as shown in Figure 2a and 3b. Initial conditions of outputs such as height of the tank 1 and tank 2 given as 12.75, 13.1 respectively.

Input-output data is generated from MATLAB code of quadruple tank system with noise which is represented in graphical forms as shown in Figure 3a and 4b. Out of 1000 samples 750 samples are taken for training data and 250 samples are taken for validation data as shown in above figure.
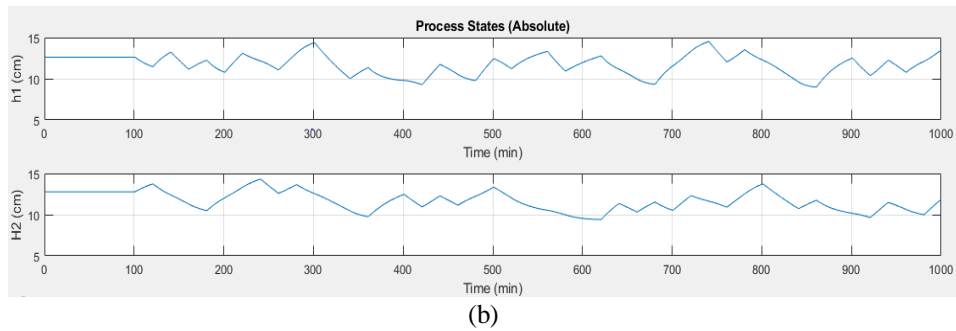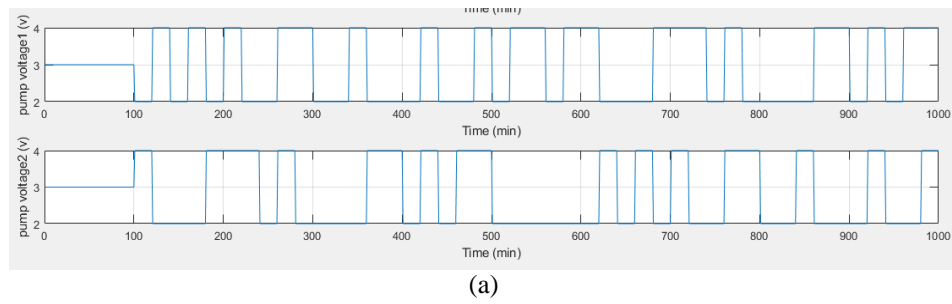
(a)

(b)

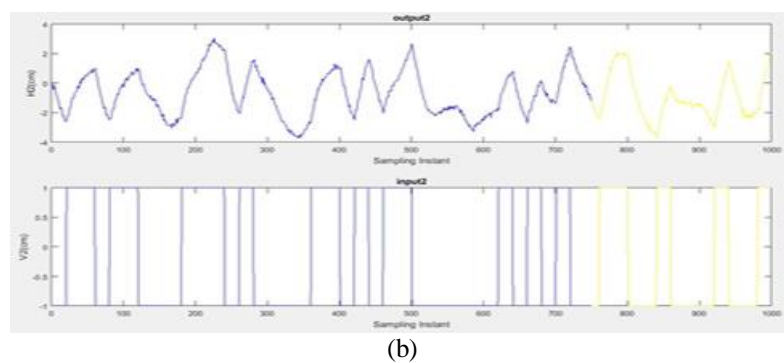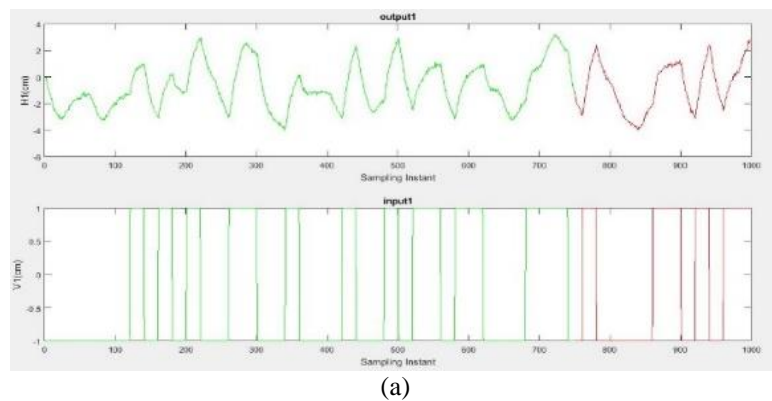Figure 2. Input and outputs plots for quadruple tank system without noise

(a)

(b)

Figure 3. Input and output plots for quadruple tank system without noise

3.2. System Identification

Using data from Figure 2a and 2b, two polynomial models are generated namely OE and ARX model as shown in Figure 4a and 4b.
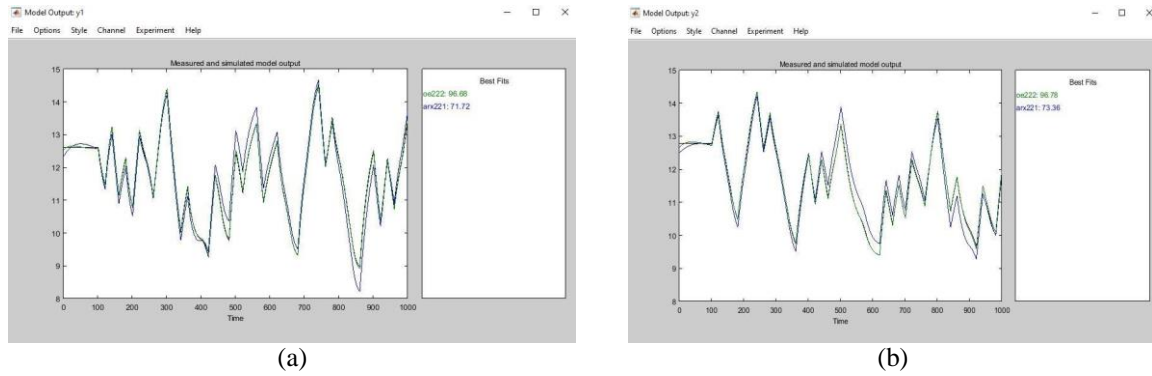
(a)   (b)

Figure 4. Fit rates for output y1 and y2 without noise

Model output y1 has fitrates of 96.88, 71.72 for OE & ARX model and model output y2 has fit rates of 96.78, 73.36 for OE and ARX without noise models. Using data from Figure 4a and 4b. two polynomial models are generated namely ARX and OE model as shown in Figure 5a, 5b, 6a and 6b.

(a)   (b)
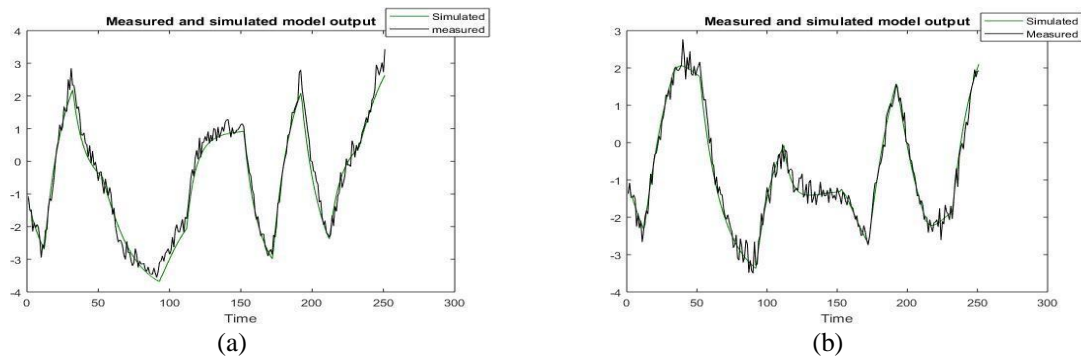
Figure 5. ARX model output y1 and y2 with noise

Discrete ARX model of four tank system is obtained as

$$\begin{bmatrix} \dfrac{0.19081z^{-1} + 0.1435z^{-2}}{1 - 0.4197z^{-1} - 0.409z^{-2}} & \dfrac{0.006292z^{-1} + 0.06521z^{-2}}{-0.05776z^{-1} - 0.0187z^{-2}} \\ \dfrac{-0.034z^{-1} + 0.04084z^{-2}}{1 - 0.4207z^{-1} - 0.4367z^{-2}} & \dfrac{0.4z^{-1} + 0.11z^{-2}}{-0.0674z^{-1} - 0.00412z^{-2}} \end{bmatrix}$$

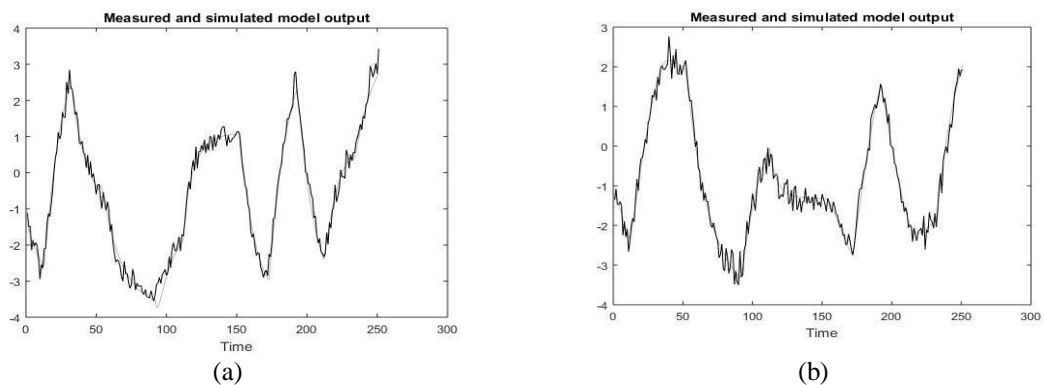(a)   (b)

Figure 6. OE model output y1 and y2 with noise

Discrete OE model of four tank system is obtained as

$$\begin{bmatrix} \dfrac{0.1081z^{-1} + 0.256z^{-2}}{1 - 0.141z^{-1} - 0.707z^{-2}} & \dfrac{-0.0817z^{-1} + 0.144z^{-2}}{1 - 1.3580z^{-1} + 0.3966z^{-2}} \\ \dfrac{0.0017z^{-1} + 0.01047z^{-2}}{1 - 1.78z^{-1} + 0.7932z^{-2}} & \dfrac{0.1492z^{-1} + 0.1397z^{-2}}{1 - 1.88z^{-1} + 0.8857z^{-2}} \end{bmatrix}$$

From the above Figure 5, model output yl has fit rates of 86.43, 82.49 and similarly from the Figure 6, model output y2 has fit rates of 85.67, 84.09 for ARX and OE noise models.

3.3. Closed Loop Response
Simulating the MPC plant model with Simulink input and output plots are generated for MPC plant without noise as shown in Figure 7 and Figure 8. Prediction and control horizons are considered as 5 and 2 respectively. Simplex optimization technique is used for optimization of MPC parameters.
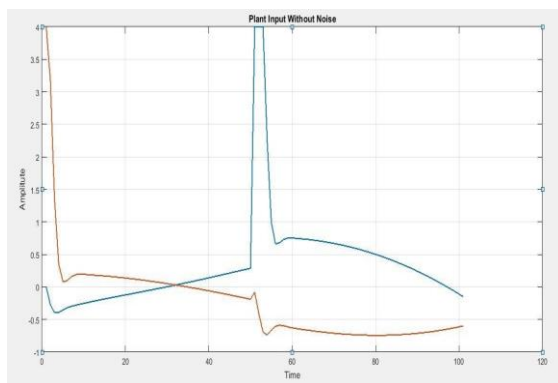

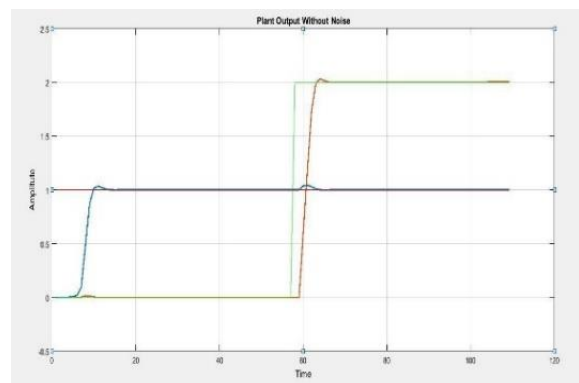Figure 7. Input plots for MPC plant without noise

Figure 8. Output plots for MPC plant without noise

From the Figure 8, outputs increase slowly with each step change i.e. outputs track the step inputs and reaches steady state. Simulating the MPC plant model with Simulink input and output plots are generated for MPC plant with noise as shown in Figure 9 and 10. From Figure 10, initially there is an inverse response for each step change and reaches steady state at the end.
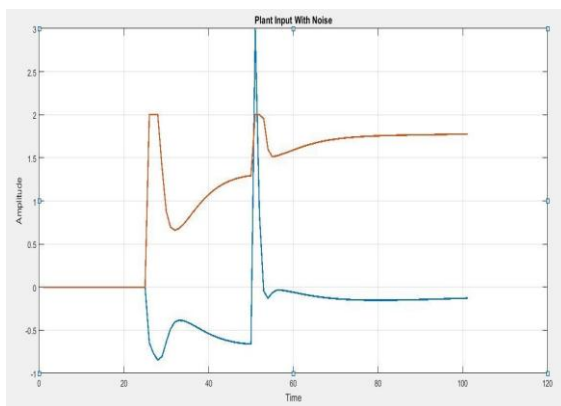

Figure 9. Input plots for MPC plant with noise
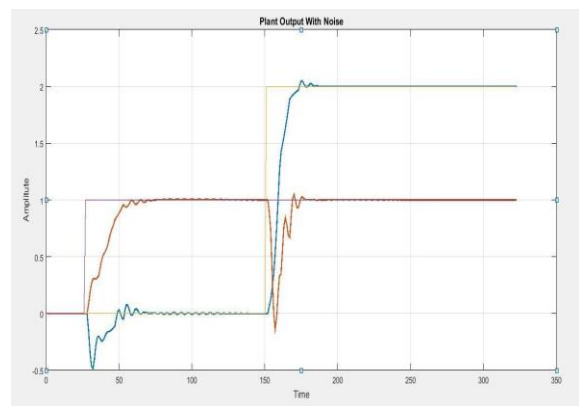

Figure 10. Output plots for MPC plant with noise

4. CONCLUSIONS
In this section, primary importance has been paid to the development of a MPC for an identified model of a Multi-Input and Multiple-Output process . A Four Tank System was considered for the generation of input-output data. This data, once generated, used for the estimation of two polynomial models, namely

ARX (Auto Regressive Exogenous) model and OE (Output-Error) model. With each of model output generated, the Fit-Rates of the models were compared to find out the most efficient model. After choosing the best model, the model equations developed. These model equations were then considered as the plant equations for developing a MPC Two sets of results were obtained after the MPC was developed and tested. One was without noise and other was with noise. Both sets of results were a success as the output signals traced the step input signals after some steady oscillations in real-time within a very short period of time, which indicated a good response time. Quite commonly MPC applications in industries involves dozens of inputs and outputs. The need is to develop multivariable models of such sizes through plant tests puts unprecedented demands on model identification techniques. In most cases, model fitting done using SISO, MISO methods. Because the model for each output is fitted separately in these methods, correlations among different outputs cannot be captured or exploited. Particularly designing a MPC in some applications where some of the controlled variable are either not measured or measured with large delays and must be inferred from secondary process measurements for satisfactory control. Many MPC's that perform well when first commissioned deteriorate over time, some leading to eventual shutdown. In order to sustain benefits of MPC's over long period of time, a mechanism to detect abnormality and dragonize its root cause is needed. The results can be communicated to engineers and can also be used to adapt the control parameters.

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